Insurance Value of Natural Capital

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Abstract. Nature-based solutions to insurance are in high demand. We explore the idea that natural capital has value insofar as a sufficiently high stock can buffer the effects of uncertain renewal. We outline a formal model that susbstantiates such claim. We propose a definition for the insurance value of natural capital for a stochastic and dynamic ecosystem that provides ecosystem services and is subject to human impacts. The insurance value of natural capital depends on the properties of ecosystem dynamics as well as on risk- and time preferences of ecosystem users. It can be positive or negative. We relate the natural insurance value to prudent use of ecosystems and precautionary investments in the natural capital stock. For the case of logarithmic utility we find that optimal management becomes more conservative with increasing uncertainty if and only if the insurance value of the natural capital stock is positive. We qualify this finding for more general forms of the intertemporal utility function.

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1 Introduction

The idea of a natural insurance value plays an important role in science and policy-making. It has been conceptualized as the reduction of the risk premium that a risk-averse ecosystem manager can obtain by conservative ecosystem management (Baumgärtner 2007, Quaas and Baumgärtner 2008). With the exemption of Augeraud-Véron et al. (2017), who study the natural insurance value of biodiversity in a dynamic model, the literature on natural insurance values has so far focused on a static analysis, ignoring that the use of ecosystems takes place in a dynamic setting. Especially when considering the value of natural capital, such a dynamic perspective is necessary, considering present and future investments and dis-investments into the natural assets. Of course, these investments are influenced by uncertainty of ecosystem dynamics. A more or less conservative management of ecosystems with increasing uncertainty is linked to the prudence (Kimball 1990) of the value function associated to the benefits derived from using the dynamic ecosystem (Kapaun and Quaas 2013). However, it has not yet been linked to the insurance value of natural capital.

In this paper we study the interaction between natural insurance value and prudent use of ecosystems in a dynamic setting. We set up a generic dynamic ecological-economic model with a risk-averse ecosystem manager who derives benefits from an uncertain flow of ecosystem services. Conservative ecosystem use resembles the dynamic self-protection problem recently studied in several contributions (Courbage and Rey 2012, Eeckhoudt et al. 2012, Wang and Li 2015): Investing in natural ecosystem capital can be seen as a 'precautionary effort' that reduces the probability of a loss event occurring, i.e. provides 'self protection' in the sense of Ehrlich and Becker (1972). This approach extends the model set-up by Baumgärtner and Strunz (2014) to a dynamic setting. We show that the propensity to use the natural insurance function of the ecosystem by means of conservative ecosystem use is not a generic outcome, but depends on the decision-maker's risk and time preferences as well as on ecosystem dynamics and processes. For the case of logarithmic utility we find that optimal management becomes more conservative with increasing uncertainty if and only if the insurance value of the natural capital stock is positive. We qualify this finding for more general forms of the intertemporal utility function.

In the next section we propose a generic model of stochastic natural capital dynamics within a harvested ecosystem and propose our definition of the natural insurance value, based on Lucas (2003). In Section 3, we show how the insurance value of natural capital is related to precautionary ecosystem management. The final section concludes.

2 Concept of Insurance Value of Natural Capital

To develop out concept of insurance value of natural capital, we proceed in two steps. First, we outline a setting to attach value to a stock of natural capital, when managed under stochasticity. Second, we define the risk premium and the insurance value of natural capital by comparison with the deterministic case.

2.1 Value of natural capital

We consider the stochastic dynamics of a natural capital (resource) stock in discrete time $t = 0, 1, 2 \ldots$. Using s_t to denote the stock at the beginning of the period $[t, t+1[$, with harvest h_t and 'escapement' $x_t = s_t - h_t$, the stock at the beginning of the next period is given by

$$s_{t+1} = \mathbf{Z}_{t+1}\, F(s_t - h_t) \tag{1}$$

with $F' > 0$, $F'' < 0$, and $\mathbf{Z}_t$ is an independently and identically distributed, multiplicative shock, with positive support and mean equal to one. The model of resource dynamics (1) follows the standard approach in considering a stochastic resource in discrete time (e.g. Reed 1979, Clark 1990, McGough et al. 2009).

The corresponding deterministic natural capital dynamics are given by (1) assuming that $\mathbf{Z}_t \equiv E[\mathbf{Z}_t] = 1$ for all t.

Utilizing the natural capital stock generates a net economic surplus $\pi(h_t, s_t)$ during period $[t, t+1[$, measured in monetary terms, where $\pi_{h_t} > 0$ and $\pi_{s_t} > 0$ (subscripts denoting partial derivatives). We think of this surplus as generating income for some ecosystem manager. Due to stochastic resource dynamics future income is uncertain.

The ecosystem manager has preferences over the uncertain steam of future income described by a dynamic expected utility function which he seeks to maximize by choosing harvest rates,

$$\widetilde{V}(s_0) = \max_{\{h_t\}} \mathbb{E}\left[\sum_{t=0}^{\infty} \delta^t\, u\left(\pi(h_t, s_t)\right)\right] \text{ subject to (1)}, \tag{2}$$

where $u(\cdot)$ is per-period utility derived from the economic surplus, and $\delta \in (0, 1)$ is the utility discount factor. In this setting, the concavity of the instantaneous utility function $u(\cdot)$ captures both the ecosystem manager's preference for intertemporal income smoothing and the degree of risk aversion. We consider a generalization that disentangles these two effects below.

While utility depends on the entire prospect of future income streams, the *maximum* expected present value of utility, $\widetilde{V}(s_0)$, is a function (only) of the initial stock of natural capital, s_0. Given the structure of the optimization problem (2), $\widetilde{V}(s_0)$ is an increasing function of s_0: A larger initial natural capital stock gives rise to a higher level of expected present value of utility.

As resource dynamics are autonomous, discounting is geometric, and the time horizon is infinite, the value $\widetilde{V}$ of the stochastic optimization problem (2) is solution of the following (implicit) stochastic dynamic programming equation

$$\widetilde{V}(s) = \max_{h} \left\{ u\left(\pi\left(h, s\right)\right) + \delta\, \mathbb{E}\left[\widetilde{V}(\mathbf{Z}\, F(s-h))\right] \right\}. \tag{3}$$

We suppose that the value function $\widetilde{V}$ in (2) is the unique solution of the above dynamic programming equation (3).

In the recursive stochastic dynamic programming formulation, we can also consider a more general type of preferences that disentangles preferences for intertemporal consumption smoothing and risk aversion. To this end, we write the Bellman equation for the infinite-time-horizon stochastic problem as

$$\widetilde{V}_{\phi}(s) = \max_{h} \left\{ u\left(\pi\left(h, s\right)\right) + \delta\, \phi^{-1}\left(\mathbb{E}\left[\phi\left(\widetilde{V}_{\phi}(\mathbf{Z}\, F(s-h))\right)\right]\right) \right\}, \tag{4}$$

where the increasing function ϕ captures the difference between preferences for intertemporal income smoothing and risk aversion in the Arrow-Pratt sense: For example, if $\phi(\cdot)$ is concave (convex), risk aversion exceeds (falls short of) the preference for intertemporal consumption smoothing. For the special case where both $u(\cdot)$ and $\phi(\cdot)$ are isoelastic, the preferences in (4) are of the Epstein and Zin (1989, 1991) form. Augeraud-Véron et al. (2017) consider this type of preferences when studying the insurance value of biodiversity. We suppose that the above dynamic programming equation (4) has a unique solution $\widetilde{V}_{\phi}$.

2.2 Risk premium

As a point of reference, we consider the problem to choose the utility-maximizing harvest rates for the deterministic resource that is described by (1) with $\mathbf{Z}_t = 1$ for all t. The corresponding deterministic optimization problem can be written as follows

$$V(s_t) = \max_{h_t} \left\{ u\left(\pi\left(h_t, s_t\right)\right) + \delta\, V(F(s_t - h_t)) \right\}, \text{ or} \tag{5a}$$

$$V(s_0) = \max_{\{h_t\}} \sum_{t=0}^{\infty} \delta^t\, u\left(\pi(h_t, s_t)\right) \text{ subject to (1) with } \mathbf{Z}_t = 1 \tag{5b}$$

Now consider the following setting: In the deterministic case, the resource manager has to sacrifice a constant fraction R of surplus from resource harvesting each period (as in Lukomska et al. 2014). His optimization problem becomes

$$W(s_0; R) = \max_{\{h_t\}} \sum_{t=0}^{\infty} \delta^t\, u\left((1-R)\,\pi\left(h_t, s_t\right)\right) \text{ subject to (1) with } \mathbf{Z}_t = 1 \tag{6}$$

We define the ad-valorem risk premium R as follows:

Definition 1*:* The ad-valorem risk premium is the constant fraction R of net economic surplus from harvesting the deterministic resource such that the resulting present value is the same as the expected present value from harvesting the stochastic resource,

$$W(s_0; R) \stackrel{!}{=} \widetilde{V}_\phi(s_0). \tag{7}$$

This definition follows the standard of measuring the welfare effect of uncertainty (Lucas 2003), and is very similar to the approach taken by Augeraud-Véron et al. (2017). It is clear from the definition that the ad-valorem risk premium will depend on the initial stock of natural capital. In the following we thus consider the ad-valorem risk premium as a function of the initial stock of natural capital, $R(s_0)$, or for short we write $R(s)$.

The ad-valorem risk premium R increases with risk as follows. Let $\overline{\mathbf{Z}}$ be a random variable which is more variable than $\mathbf{Z}$ in the increasing and concave order sense. Suppose that the value function $\widetilde{V}_\phi$ in (4) is increasing and concave. Then, it can easily be deduced from (4) that, with obvious notations, $\overline{\widetilde{V}}_\phi \leq \widetilde{V}_\phi$. As $W(s_0; R)$ decreases with R by (6), we conclude that, with obvious notations, $\overline{R} \geq R$.

2.3 Insurance value of natural capital

We are interested in the question whether a larger stock of natural capital will decrease or increase the risk premium. Following the literature on natural insurance values, natural capital has a natural insurance function if and only if an increase of the stock decreases the risk premium (Baumgärtner 2007, Quaas and Baumgärtner 2008, Baumgärtner and

Strunz 2014).

Based on the definition of the ad-valorem risk premium, we thus define the insurance value of natural capital as follows.

Definition 2: The insurance value of the natural capital stock s_0 is defined as

$$I(s_0) = -\frac{dR(s_0)}{ds_0}. \tag{8}$$

We thus define the insurance value not in terms of monetary value, but in terms of the percentage of future income value that the decision maker is willing to sacrifice in order to get rid of uncertainty.

As an analogue, consider the Nato states that have agreed to spend two percent of GDP on military purposes (not all Nato members comply with this agreement). If the world would get safer such that this two percent target could be reduced, the amount of reduction of this fraction of GDP would be the corresponding insurance value according to Definition 2.

3 Insurance value of natural capital: Implications for resource management

We now turn to the question how the insurance value of natural capital is linked to optimizing resource management. First, we provide an expression of the insurance value. Second, we derive implications for resource management.

3.1 Insurance value of natural capital for isoelastic utility

In this section, we focus on the standard assumption of an isoelastic utility function

$$u(\pi) = \begin{cases} \frac{\pi^{1-\eta}-1}{1-\eta} & \text{for } \eta > 0, \eta \neq 1 \\ \ln(\pi) & \text{for } \eta = 1 \end{cases} \tag{9}$$

Let us introduce a new function

$$\Phi(x) = \phi\left(\frac{x^{1-\eta}-1}{(1-\delta)(1-\eta)}\right), \tag{10}$$

which is increasing and concave, as ϕ is increasing and concave.

For the specification (9) of the per-period utility function, the Bellman equation (4) has a solution given by (see Appendix A)

$$\widetilde{V}_\phi = \phi^{-1} \circ \Phi \circ \widetilde{U}_\Phi \tag{11a}$$

$$\widetilde{U}_\Phi(s) = \max_h \left((1-\delta)\, \pi(h,s)^{1-\eta} + \delta \left(\Phi^{-1} \left(\mathbb{E} \left[\Phi \left(\widetilde{U}_\Phi \left(\mathbf{Z}\, F(s-h) \right) \right) \right] \right) \right)^{1-\eta} \right)^{\frac{1}{1-\eta}} . \tag{11b}$$

By taking $\mathbf{Z} = 1$, we immediately obtain that the Bellman equation (5) of the corresponding deterministic problem has a solution given by

$$V = \phi^{-1} \circ \Phi \circ U \tag{12a}$$

$$U(s) = \max_h \left((1-\delta)\, \pi(h,s)^{1-\eta} + \delta\, \left(U\left(F(s-h) \right) \right)^{1-\eta} \right)^{\frac{1}{1-\eta}} . \tag{12b}$$

Following Definition (7), the ad-valorem risk premium is defined implicitely by the relations

$$W(R,s) = \max_h \left((1-\delta)\, \left((1-R)\, \pi(h,s) \right)^{1-\eta} + \delta\, \left(W\left(R, F(s-h) \right) \right)^{1-\eta} \right)^{\frac{1}{1-\eta}} \tag{13a}$$

$$W(R,s) \stackrel{!}{=} \widetilde{U}_\Phi(s). \tag{13b}$$

With this set-up, and especially the iso-elastic utility function, we obtain the following results on the ad-valorem risk premium and optimal management.

Proposition 1*:* For the isoelastic utility function (9), we have

1. $W(R,s) = (1-R)\, U(s)$.
2. $h^\star(s)$ does not depend on R.
3. The ad-valorem risk premium is

$$R(s) = 1 - \frac{\widetilde{U}_\Phi(s)}{U(s)}. \tag{14}$$

Proof. see Appendix B. □

3.2 Implications of insurance value for resource management

From Proposition 1, we can now deduce implications for optimal resource management.

For the utility function (9) we can differentiate $\widetilde{U}_\Phi(s) = (1 - R(s))\,U(s)$ with respect to s and obtain

$$\widetilde{U}'_\Phi(s) = I(s)\,U(s) + (1 - R(s))\,U'(s) \tag{15}$$

dividing by $\widetilde{U}_\Phi(s)$, and using $\widetilde{U}_\Phi(s) = (1 - R(s))\,U(s)$, which follows from (13b) and Proposition 1.1, we get

$$\frac{\widetilde{U}'_\Phi(s)}{\widetilde{U}_\Phi(s)} = \frac{I(s)}{1 - R(s)} + \frac{U'(s)}{U(s)} \tag{16}$$

As $U(s) > 0$, equation (15) implies that

$$I(s) \gtreqless 0 \quad \Leftrightarrow \quad \widetilde{U}'_\Phi(s) \gtreqless (1 - R(s))\,U'(s). \tag{17}$$

That means, the insurance value of natural capital is positive if and only if the shadow price of the natural capital stock in the stochastic setting is larger than $1 - R(s)$ times the shadow price of the natural capital stock in the corresponding deterministic setting.

For the next step, we consider the economic surplus as a function of stock s_t and escapement $x_t = s_t - h_t$ by

$$\Pi(x_t, s_t) \equiv \pi(s_t - x_t, s_t). \tag{18}$$

Using $\widetilde{x}^\star(s)$ to denote the optimal escapement, assuming an interior solution, the Bellman equation (11b) can be written as

$$\widetilde{U}_\Phi(s)^{1-\eta} = (1 - \delta)\,\Pi(\widetilde{x}^\star(s), s)^{1-\eta} + \delta\,\left(\Phi^{-1}\left(\mathbb{E}\left[\Phi\left(\widetilde{U}_\Phi\left(\mathbf{Z}\,F(\widetilde{x}^\star(s))\right)\right)\right]\right)\right)^{1-\eta}. \tag{3'''}$$

Differentiating with respect to s, using the envelope theorem, we get

$$\widetilde{U}_\Phi(s)^{-\eta}\,\widetilde{U}'_\Phi(s) = (1 - \delta)\,\Pi\left(\widetilde{x}^\star(s), s\right)^{-\eta}\,\Pi_s\left(\widetilde{x}^\star(s), s\right), \tag{19}$$

as the value in the future depends only on x.

Similarly, using $x^\star(s)$ to denote the optimal escapement for the deterministic problem, we obtain

$$U(s)^{-\eta}\,U'(s) = (1 - \delta)\,\Pi\left(x^\star(s), s\right)^{-\eta}\,\Pi_s\left(x^\star(s), s\right). \tag{20}$$

Management under uncertainty is more conservative than in the deterministic case if (and only if) $\widetilde{x}^\star(s) > x^\star(s)$. This means, that at a given initial stock level s the manager would leave a larger escapement level when facing uncertainty about biological productivity of the stock than in a setting where the future stock size is deterministic. More conservative management under uncertainty can also be interpreted as precautionary saving into the natural capital stock, and is optimal if the value function of the dynamic optimization problem exhibits *prudence* (Kapaun and Quaas 2013).

We can relate these results to the insurance value of natural capital by using (17) in the following formulation

$$I(s) \gtreqless 0 \quad \Leftrightarrow \quad \Pi\left(\widetilde{x}^*(s), s\right)^{-\eta}\, \Pi_s\left(\widetilde{x}^*(s), s\right) \gtreqless \left(1 - R(s)\right)^{1-\eta}\, \Pi\left(x^*(s), s\right)^{-\eta}\, \Pi_s\left(x^*(s), s\right) \tag{21}$$

This result allows to relate the insurance value of natural capital – $I(s)$ – to the escapement in the stochastic vs. the deterministic settings.

For the case of logarithmic utility, we obtain the following result

Proposition 2*:* For the case of logarithmic utility, (9) with $\eta = 1$, the insurance value of natural capital is positive if and only if management is more conservative in the stochastic than in the deterministic setting, i.e.

$$\eta = 1: \quad I(s) \gtreqless 0 \quad \Leftrightarrow \quad \widetilde{x}^\star(s) \gtreqless x^\star(s). \tag{22}$$

Proof. See Appendix C. □

In case of logarithmic utility, a positive insurance value is a necessary and sufficient condition for precautionary savings in the natural capital stock. Note that logarithmic utility is a particularly relevant case, as this is the type of utility considered in most macroeconomic models, also in the domains of natural resource use (Levhari and Mirman 1980) and climate change (Golosov et al. 2014, Gerlagh and Liski 2018).

In the case of an isoelastic utility, with an elasticity different from one, the relationship between precautionary resource use and the insurance value of natural capital is a little more involved. As $0 < 1 - R(s) < 1$, we obtain the following results.

Proposition 3*:* Assume isoelastic utility (9).

1. For $\eta < 1$, (a) a positive insurance value is a necessary, but not sufficient, condition for precautionary investment in the natural capital stock, and (b) a negative insurance value is a sufficient, but not necessary, condition for precautionary investment

in the natural capital stock,

$$\eta < 1: \qquad \begin{aligned} I(s) > 0 &\Leftarrow \widetilde{x}^\star(s) > x^\star(s) \\ I(s) < 0 &\Rightarrow \widetilde{x}^\star(s) < x^\star(s) \end{aligned} \tag{23a}$$

2. For $\eta > 1$, (a) a positive insurance value is a sufficient, but not necessary, condition for precautionary investment in the natural capital stock, and (b) a negative insurance value is a necessary, but not sufficient, condition for precautionary investment in the natural capital stock,

$$\eta < 1: \qquad \begin{aligned} I(s) > 0 &\Rightarrow \widetilde{x}^\star(s) > x^\star(s) \\ I(s) < 0 &\Leftarrow \widetilde{x}^\star(s) < x^\star(s) \end{aligned} \tag{23b}$$

Proof. See Appendix C. □

The case $\eta < 1$ means that incomes at different points in time are relatively close substitutes, while $\eta > 1$ means that incomes at different points in time are relatively close complements. In both cases, it matters for the decision-maker that the value-added risk premium reduces future incomes in the deterministic setting, but in a different way. For intertemporal substitutes, $\eta < 1$, the decision maker tends to compensate future loss of income by increasing present incomes. This effect increases harvest in the deterministic setting, relative to the stochastic setting. Thus, a positive insurance value is not sufficient for precautionary investment in the natural capital stock.

For intertemporal complements, $\eta > 1$, the decision maker tends to compensate future loss of income by investing in the natural capital stock and reducing present incomes. This effect decreases harvest in the deterministic setting, relative to the stochastic setting. Thus, even for a negative insurance value the decision maker may still choose precautionary investment in the natural capital stock.

4 Conclusions

In this paper we have proposed a definition for the insurance value of natural capital for a stochastic and dynamic ecosystem that provides ecosystem services and is subject to human impacts. We have shown that the insurance value of natural capital can be positive or negative, depending on the properties of ecosystem dynamics as well as on risk- and time preferences of ecosystem users. The sign and level of the insurance value are related to prudent use of ecosystems and precautionary investments in the natural capital stock. For the case of logarithmic utility we have shown that optimal management becomes more conservative with increasing uncertainty if and only if the insurance value of the natural capital stock is positive. For more general forms of the intertemporal utility function, this finding is qualified, and a positive insurance value can be either a positive or a sufficient condition for precautionary ecosystem management, but generally not both.

The concept and model proposed in this paper may form the basis for next steps of quantifying and analyzing the insurance value of natural capital and management implications. Using numerical methods of stochastic programming or stochastic dynamic programming (e.g., value function iteration), it is a straightforward task to quantify the insurance value for natural capital such as fish stocks (McGough et al. 2009, Kapaun and Quaas 2013, Tahvonen et al. 2017), rangelands (Perrings and Walker 1997, Janssen et al. 2004, Quaas et al. 2007, Quaas and Baumgärtner 2012), or water (Walker et al. 2010). As previous studies most often find that uncertainty induces more conservative management of ecosystems, we expect that most of these natural capital stocks have a positive natural insurance value.

Appendix

A Bellman equation (11b) for isoelastic utility

In the following, we assume $\eta \neq 1$ in the isoelastic utility (9). Then, we show at the end that the result (11b) that we obtain is well-defined also for $\eta = 1$. With the specification (9), the Bellman equation (4) becomes

$$\widetilde{V}_\phi(s) = \max_h \left\{ \frac{\pi(h,s)^{1-\eta} - 1}{1-\eta} + \delta\, \phi^{-1} \left(\mathbb{E}\left[\phi\left(\widetilde{V}_\phi\left(\mathbf{Z}\, F(s-h) \right) \right) \right] \right) \right\}, \tag{24}$$

Multiplying by $(1-\delta)\,(1-\eta)$ gives

$$\begin{aligned} (1-\delta)\,(1-\eta)\,\widetilde{V}_\phi(s) = \max_h \Bigg\{ &(1-\delta)\,\pi(h,s)^{1-\eta} - (1-\delta) \\ &+ \delta\,(1-\delta)\,(1-\eta)\,\phi^{-1}\left(\mathbb{E}\left[\phi\left(\widetilde{V}_\phi\left(\mathbf{Z}\, F(s-h) \right) \right) \right] \right) \Bigg\}. \end{aligned} \tag{25}$$

Further rearranging, we obtain that

$$\left((1-\delta)\,(1-\eta)\,\widetilde{V}_\phi(s) + 1 \right)^{\frac{1}{1-\eta}} \tag{26}$$

$$= \max_h \left((1-\delta)\,\pi(h,s)^{1-\eta} + \delta + \delta\,(1-\delta)\,(1-\eta)\,\phi^{-1}\left(\mathbb{E}\left[\phi\left(\widetilde{V}_\phi\left(\mathbf{Z}\,F(s-h)\right)\right)\right]\right)\right)^{\frac{1}{1-\eta}} \tag{27}$$

$$= \max_h \left((1-\delta)\,\pi(h,s)^{1-\eta} + \delta \left(\Phi^{-1}\left(\mathbb{E}\left[\phi\left(\widetilde{V}_\phi\left(\mathbf{Z}\,F(s-h)\right)\right)\right]\right)\right)^{1-\eta}\right)^{\frac{1}{1-\eta}}, \tag{28}$$

since the functions ϕ and Φ in (10) are related by

$$\Phi\,(x) = \phi\left(\frac{x^{1-\eta}-1}{(1-\delta)\,(1-\eta)} \right) \text{ and } \left(\Phi^{-1}(y)\right)^{1-\eta} = (1-\delta)\,(1-\eta)\,\phi^{-1}(y) + 1. \tag{29}$$

Therefore, if we define a new function $\widetilde{U}_\Phi$ as in (11a) by $\widetilde{U}_\Phi(s) = \Phi^{-1}(\phi(\widetilde{V}_\phi(s)))$, we easily obtain that $\widetilde{U}_\Phi(s) = \left((1-\delta)\,(1-\eta)\,\widetilde{V}_\phi(s) + 1 \right)^{\frac{1}{1-\eta}}$, and that $\Phi(\widetilde{U}_\Phi(s)) = \phi(\widetilde{V}_\phi(s))$. Overall, the new function $\widetilde{U}_\Phi$ is solution of

$$\widetilde{U}_\Phi(s) = \max_h \left((1-\delta)\,\pi(h,s)^{1-\eta} + \delta\,\left(\Phi^{-1}\left(\mathbb{E}\left[\Phi\left(\widetilde{U}_\Phi\left(\mathbf{Z}\,F(s-h)\right)\right)\right]\right)\right)^{1-\eta}\right)^{\frac{1}{1-\eta}}. \tag{30}$$

B Proof of Proposition 1

1. Rearranging (13b), we get

$$\left(\frac{W(R,s)}{1-R} \right)^{1-\eta} = \max_h \left((1-\delta)\,\pi(h,s)^{1-\eta} + \delta \left(\frac{W\left(R, F(s-h)\right)}{1-R} \right)^{1-\eta} \right). \tag{31}$$

   Comparing with (12b), we obtain that $W(R,s) = (1-R(s))\,U(s)$ by uniqueness of the solutions of the Bellman equation (12b) (because uniqueness holds for the Bellman equation (4)).

2. Using $W(R,s) = (1-R(s))\,U(s)$ in (31), $R(s)$ cancels out of the Bellman equation and thus h does not depend on R.

3. From the definition (13b) of the risk premium R, namely $\widetilde{U}_\Phi(s) = W(R,s)$ and from $W(R,s) = (1-R(s))\,U(s)$, we deduce that $\widetilde{U}_\Phi(s) = (1-R(s))\,U(s)$. Rearranging leads to (14).

C Proof of Propositions 2–3

We have

$$\frac{d}{dx}\left(\Pi(x,s)^{-\eta}\,\Pi_s(x,s)\right) = \underbrace{-\eta\,\Pi(x,s)^{-\eta-1}}_{<0}\,\underbrace{\Pi_x(x,s)}_{\leq 0}\,\underbrace{\Pi_s(x,s)}_{\geq 0} + \underbrace{\Pi(x,s)^{-\eta}}_{>0}\,\underbrace{\Pi_{sx}(x,s)}_{\geq 0} \geq 0 \qquad (32)$$

Thus, for a given level of s, and for $\eta = 1$, the left-hand side of (21) is larger (equal to, smaller) than the right-hand side whenever the argument of the function $\Pi(x,s)^{-\eta}\,\Pi_s(x,s)$ is larger (equal to, smaller) on the left-hand side than on the right-hand side. This proves Proposition 2.

A similar argument proves Proposition 3, noting that $(1-R)^{1-\eta} < 1$ for $\eta < 1$ and $(1-R)^{1-\eta} > 1$ for $\eta > 1$, as $0 < 1 - R < 1$.

References

Augeraud-Véron, E., Fabbri, G., Schubert, K., 2017. The value of biodiversity as an insurance device. AMSE Working Papers 1709, Aix-Marseille School of Economics, Marseille, France. URL https://ideas.repec.org/p/aim/wpaimx/1709.html

Baumgärtner, S., 2007. The insurance value of biodiversity in the provision of ecosystem services. Natural Resource Modeling 20, 87–127.

Baumgärtner, S., Strunz, S., 2014. The economic insurance value of ecosystem resilience. Ecological Economics 101, 21–32.

Clark, C. W., 1990. Mathematical Bioeconomics, 2nd Edition. Wiley, New York.

Courbage, C., Rey, B., 2012. Optimal prevention and other risks in a two-period model. Mathematical Social Sciences 63 (3), 213 – 217.

Eeckhoudt, L., Huang, R. J., Tzeng, L. Y., 2012. Precautionary effort: A new look. The Journal of Risk and Insurance 79 (2), 585–590.

Ehrlich, J., Becker, G. S., 1972. Market insurance, self-insurance and self-protection. Journal of Political Economy 80, 623–648.

Epstein, L. G., Zin, S., 1989. Substitution, risk aversion, and the temporal behavior of consumption and asset returns: A theoretical framework. Econometrica 57 (4), 937–969.

Epstein, L. G., Zin, S., 1991. Substitution, risk aversion, and the temporal behavior of consumption and asset returns: An empirical analysis. Journal of Political Economy 99 (2), 263–286.

Gerlagh, R., Liski, M., 2018. Consistent climate policies. Journal of the European Economic Association 16 (1), 1–44.

Golosov, M., Hassler, J., Krusell, P., Tsyvinski, A., 2014. Optimal taxes on fossil fuel in general equilibrium. Econometrica 82 (1), 41–88.

Janssen, M. A., Anderies, J. M., Walker, B. H., 2004. Robust strategies for managing rangelands with multiple stable attractors. Journal of Environmental Economics and Management 47, 140–162.

Kapaun, U., Quaas, M. F., 2013. Does the optimal size of a fish stock increase with environmental uncertainties? Environmental and Resource Economics 54 (2), 293–310.

Kimball, M., 1990. Precautionary saving in the small and in the large. Econometrica 58 (1), 53–73.

Levhari, D., Mirman, L. J., 1980. The great fish war: An example using a dynamic cournot-nash solution. The Bell Journal of Economics 11 (1), 322–334.

Lucas, Robert E., J., March 2003. Macroeconomic priorities. American Economic Review 93 (1), 1–14.

Lukomska, N., Quaas, M., Baumgärtner, S., 2014. Bush encroachment control and risk management in semi-arid rangelands. Journal of Environmental Management 145 (1), 24–34.

McGough, B., Plantinga, A. J., Costello, C., 2009. Optimally managing a stochastic renewable resource under general economic conditions. The B.E. Journal of Economic Analysis & Policy 9 (1, Contributions), Article 56.

Perrings, C., Walker, B., 1997. Biodiversity, resilience and the control of ecological-economic systems: the case of fire-driven rangelands. Ecological Economics 22, 73–83.

Quaas, M. F., Baumgärtner, S., 2008. Natural vs. financial insurance in the management of public-good ecosystems. Ecological Economics 65, 397–406.

Quaas, M. F., Baumgärtner, S., 2012. Optimal grazing management rules in semi-arid rangelands with uncertain rainfall. Natural Resource Modeling 25 (2), 364–387.

Quaas, M. F., Baumgärtner, S., Becker, C., Frank, K., Müller, B., 2007. Uncertainty and sustainability in the management of rangelands. Ecological Economics 62, 213–234.

Reed, W. J., 1979. Optimal escapement levels in stochastic and deterministic harvesting models. Journal of Environmental Economics and Management 6, 350–363.

Tahvonen, O., Quaas, M. F., Voss, R., 2017. Harvesting selectivity and stochastic recruitment in economic models of age-structured fisheries. Journal of Environmental Economics and ManagementDoi 10.1016/j.jeem.2017.08.011.

Walker, B., Pearson, L., Harris, M., Maler, K.-G., Li, C.-Z., Biggs, R., Baynes, T., 2010. Incorporating resilience in the assessment of inclusive wealth: An example from south east australia. Environmental and Resource Economics 45 (2), 183–202.

Wang, J., Li, J., 2015. Precautionary effort: Another trait for prudence. Journal of Risk and Insurance 82 (4), 977–983.